

$$\frac{dx}{dt} \cdot \frac{dt}{dy}$$

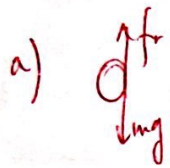
$$m = 1 \cdot 10^{-11} \text{ Kg}$$

$$\vec{r} = -bv$$

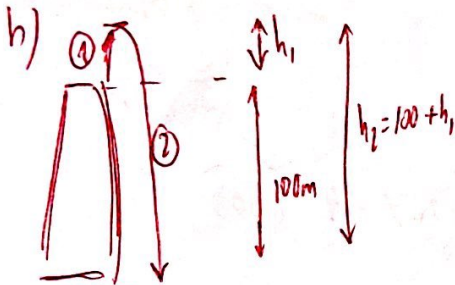
$$b = 3 \cdot 10^{-9} \text{ N/(m/s)}^2$$

n) v_{lm} ?

b) Tal snello in $h = 100 \text{ m}$
 con $v_0 = 2 \text{ m/s}$



$$\rightarrow mg - f = 0 ; mg - bv_{lm} = 0 ; v_{lm} = \frac{mg}{b} = \frac{1 \cdot 10^{-11} \cdot 9.81}{3 \cdot 10^{-9}} = 0.0327 \text{ m/s}$$



$$\tau = \frac{m}{b} = \frac{1 \cdot 10^{-11}}{3 \cdot 10^{-9}} = 0.003$$

① Solida



$$\Sigma F = m \cdot a$$

$$m \cdot a = -mg - bv$$

$$m \cdot \frac{dv}{dt} = -b(v + v_{lm})$$

$$\frac{dv}{v + v_{lm}} = \frac{-b}{m} dt$$

$$\ln(v + v_{lm}) \Big|_{v_0}^{v_t} = \ln e^{-t/\tau}$$

$$\frac{v_t + v_{lm}}{v_0 + v_{lm}} = e^{-t/\tau}$$

$$v_t = (v_0 + v_{lm})(e^{-t/\tau}) - v_{lm}$$

→ Isolamos t .

$$\ln \frac{v_t + v_{lm}}{v_0 + v_{lm}} = \frac{-b}{m} t$$

$$t = -\frac{m}{b} \ln \frac{v_t + v_{lm}}{v_0 + v_{lm}} = -0.003 \ln \frac{0 + 0.0327}{2 + 0.0327}$$

$$\boxed{t = 0.01375 \text{ s}}$$

$$\frac{dy}{dt} = (v_0 + v_{lm})(e^{-t/\tau}) - v_{lm}$$

$$dy = \int (v_0 + v_{lm})(e^{-t/\tau}) dt - \int v_{lm} dt$$

$$y = +\tau (v_0 + v_{lm})(1 - e^{-t/\tau}) - v_{lm} t$$

$$y_1 = 0.003 (2 + 0.0327)(1 - e^{(-0.01375/0.003)}) - (0.0327)(0.01375)$$

$$\boxed{y = 0.00615 \text{ m}}$$

② Bajada.



$$\Sigma F = m \cdot a$$

$$m \cdot a = mg - bv$$

$$m \cdot \frac{dv}{dt} = -b(v - v_{lm})$$

$$\frac{dv}{v - v_{lm}} = \frac{-b}{m} dt$$

$$\ln \frac{v - v_{lm}}{v_0 - v_{lm}} = -\frac{t}{\tau}$$

$$\frac{v - v_{lm}}{v_0 - v_{lm}} = e^{-t/\tau}$$

$$v = (v_0 - v_{lm})(e^{-t/\tau}) + v_{lm}$$

$$\frac{dy}{dt} = (v_0 - v_{lm})(e^{-t/\tau}) + v_{lm}$$

$$dy = \int (v_0 - v_{lm})(e^{-t/\tau}) dt + \int v_{lm} dt$$

$$y = \tau(v_0 - v_{lm})(1 - e^{-t/\tau}) + v_{lm}t$$

Alcanza v_{lm} en $t = 4\tau$

$$y = 0.003(0 - 0.0327)(1 - e^{-4/\tau}) + 0.0327 \cdot 4 \cdot 0.003$$

$$y = 0.000328 \text{ m.}$$

$$y_{\text{restante}} = 100 + 0.00615 - 0.000328 = 100.0058 \text{ m.}$$

$$v = \frac{y}{t} \quad ; \quad t = \frac{y}{v} = \frac{100.0058}{0.0327} = 3058.28 \text{ s.}$$

$$t_1 = 0.01375 \text{ s}$$

$$t_2 = 3058.28 + 4\tau = 3058.29 \text{ s}$$

$$t_{\text{TOT}} = t_1 + t_2 = 3058.307 \text{ s}$$

$$t = 50' 57 \text{ s } \approx 515.$$

Problema 2. $\Rightarrow$ Demostración movimiento cuadrático hacia abajo.

$$\Sigma F = m \cdot a$$

$$m \cdot a = mg - cv^2$$

$$m \cdot \frac{dv}{dt} = mg - cv^2$$

$$\frac{dv}{dt} = \frac{mg}{m} - \frac{cv^2}{m}$$

$$\frac{dv}{dt} = g \left(1 - \frac{cv^2}{mg} \right)$$

$$\frac{dv}{dt} = g \left(1 - \frac{v^2}{v_{lm}^2} \right)$$

$$\frac{dv}{\left(1 - \frac{v^2}{v_{lm}^2} \right)} = g dt$$

$$\int \frac{dv}{\left(1 - \left(\frac{v}{v_{lm}} \right)^2 \right)} = \int g dt$$

cambio de variable:

$$\frac{v}{v_{lm}} = u$$

$$dv = v_{lm} \cdot du$$

$$\int \frac{v_{lm} \cdot du}{1 - u^2} = \operatorname{arctg} h(u) v_{lm}$$

$$\operatorname{arctg} h \left(\frac{v(t)}{v_{lm}} \right) \cdot v_{lm} \Big|_0^t = gt$$

$$\operatorname{arctg} h \left(\frac{v(t)}{v_{lm}} \right) = gt / v_{lm}$$

$$\operatorname{tg} h \left(\frac{gt}{v_{lm}} \right) = \frac{v(t)}{v_{lm}}$$

$$v(t) = v_{lm} \cdot \operatorname{tg} h \left(\frac{gt}{v_{lm}} \right)$$

$$\frac{dy}{dt} = v(t), \quad dy = v(t) \cdot dt$$

Supongo $y_0 = 0$

$$\int dy = \int_0^t v_{lm} \cdot \operatorname{tg} h \left(\frac{gt}{v_{lm}} \right) dt ;$$

$$y(t) = \int_0^t v_{lm} \cdot \frac{\operatorname{sen} h \left(\frac{gt}{v_{lm}} \right)}{\operatorname{cos} h \left(\frac{gt}{v_{lm}} \right)} \cdot dt$$

$$\begin{cases} \frac{gt}{v_{lm}} = u \\ \frac{dt}{dt} = \frac{v_{lm}}{g} du \end{cases} \quad \int v_{lm} \cdot \frac{\operatorname{sen} h(u)}{\operatorname{cos} h(u)} \cdot \frac{v_{lm}}{g} du$$

$$y(t) = \frac{v_{lm}^2}{g} \cdot \int \frac{\operatorname{sen} h u}{\operatorname{cos} h u} \cdot du = \frac{v_{lm}^2}{g} \cdot \ln (\operatorname{cos} h u)$$

$$y(t) = \frac{v_{lm}^2}{g} \cdot \ln (\operatorname{cos} h \left(\frac{gt}{v_{lm}} \right)) \Big|_0^t$$

$$y(t) = \frac{v_{lm}^2}{g} \left(\ln (\operatorname{cos} h \left(\frac{gt}{v_{lm}} \right)) - 0 \right)$$

$$y(t) = \frac{v_{lm}^2}{g} \cdot \ln (\operatorname{cos} h \left(\frac{gt}{v_{lm}} \right))$$

⊗ Aprender.

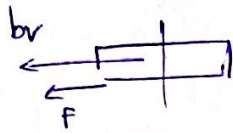
1/ FN LINEAL + F

$$\mu = 0.2$$

ρ (plomo)

$$\beta, \gamma$$

$$v_0 = 10 \text{ m/s}$$



$$\Sigma F = m \cdot a$$

$$m \cdot a = -bv - F$$

$$m \cdot \frac{dv}{dt} = -bv - F$$

$$m \cdot \frac{dv}{dt} = -b \left(v + \frac{F}{b} \right)$$

$$\int \frac{dv}{\left(v + \frac{F}{b} \right)} = \int \frac{-b}{m} dt$$

cambio variable:

$$\frac{F}{b} + v = u$$

$$dv = du$$

$$\int \frac{du}{u} = \ln(u)$$

$$\Rightarrow \ln \left(\frac{F}{b} + v \right) \Big|_{v_0}^{vt} = -\frac{t}{\tau} ; \quad \frac{\frac{F}{b} + vt}{\frac{F}{b} + v_0} = e^{-t/\tau} ;$$

$$; \quad \frac{F}{b} + vt = \left(\frac{F}{b} + v_0 \right) e^{-t/\tau} ; \quad vt = \left(\frac{F}{b} + v_0 \right) e^{-t/\tau} - \frac{F}{b}$$

$$\left[vt = (v_0 \cdot e^{-t/\tau}) + \frac{F}{b} (e^{-t/\tau} - 1) \right]$$

$$F = \mu \cdot N ; \quad F = \mu \cdot m \cdot g ; \quad \rho = \frac{m}{V} ; \quad m = \rho \cdot V$$

$$F = \mu \cdot \rho \cdot V \cdot g \Rightarrow F = \mu \cdot \rho \cdot \frac{4}{3} \pi R^3 \cdot g$$

$$v(t) = \frac{dy}{dt} ; \quad dy = v(t) \cdot dt ; \quad y_0 = 0$$

$$\frac{dy}{dt} = \left(\frac{F}{b} + v_0 \right) e^{-t/\tau} - \frac{F}{b}$$

$$dy = \left(\frac{F}{b} + v_0 \right) \int e^{-t/\tau} dt - \int \frac{F}{b} dt$$

$$dy = \left(\frac{F}{b} + v_0 \right) (-\tau) (e^{-t/\tau} - 1) - \frac{F}{b} t$$

$$dy = \left(\frac{F}{b} + v_0 \right) (\tau) (1 - e^{-t/\tau}) - \frac{F}{b} t$$

LINEAL CON ROZAMIENTO

⊗ Importante:

$$\Sigma F = m \cdot a$$

$$m \cdot \frac{dv}{dt} = -bv$$

$$\int \frac{dv}{v} = \int \frac{-b}{m} dt$$

$$\ln v \Big|_{v_0}^{vt} = \frac{-b}{m} t$$

$$\ln \frac{vt}{v_0} = -\frac{t}{\tau}$$

$$\frac{vt}{v_0} = e^{-t/\tau}$$

$$vt = v_0 \cdot e^{-t/\tau}$$

$$\frac{dx}{dt} = v_0 \cdot e^{-t/\tau}$$

$$dx \Big|_{x_0}^x = v_0 \cdot \int e^{-t/\tau} dt$$

$$x - x_0 = v_0 (-\tau) \int e^{-t/\tau} \cdot \frac{-1}{\tau} dt$$

$$x = x_0 - \tau v_0 (e^{-t/\tau} - 1)$$

Horizontal

$$x = x_0 - \tau (F/b + v_0) (e^{-t/\tau} - 1) - F/b t$$

$$\Sigma F = m \cdot a$$

$$m \cdot \frac{dv}{dt} = -bv - F$$

$$m \cdot \frac{dv}{dt} = -b(v + \frac{F}{b})$$

$$\int \frac{dv}{v + \frac{F}{b}} = \int \frac{-b}{m} dt$$

$$dv = du$$

$$u = v + \frac{F}{b}$$

$$\ln \frac{F/b + v}{F/b + vt} \Big|_{v_0}^{vt} = -\frac{t}{\tau}$$

$$\frac{F/b + vt}{F/b + v_0} = e^{-t/\tau}$$

$$vt = (F/b + v_0) (e^{-t/\tau}) - \frac{F}{b}$$

$$\frac{dx}{dt} = (F/b + v_0) (e^{-t/\tau}) - \frac{F}{b}$$

$$dx \Big|_{x_0}^x = (F/b + v_0) \int e^{-t/\tau} dt - \int \frac{F}{b} dt$$

$$x - x_0 = -\tau (F/b + v_0) \int e^{-t/\tau} \cdot \frac{-1}{\tau} dt - \frac{F}{b} t$$

Horizontal + Free

$$\Sigma F = m \cdot a$$

$$m \cdot \frac{dv}{dt} = -mg - bv$$

$$m \cdot \frac{dv}{dt} = -b(v + \frac{mg}{b})$$

$$\int \frac{dv}{v + \frac{mg}{b}} = \int \frac{-b}{m} dt$$

$$\ln v + \frac{mg}{b} \Big|_{v_0}^{vt} = \frac{-b}{m} t$$

$$\frac{vt + \frac{mg}{b}}{v_0 + \frac{mg}{b}} = e^{-t/\tau}$$

$$vt = (v_0 + \frac{mg}{b}) (e^{-t/\tau}) - \frac{mg}{b}$$

$$\frac{dy}{dt} = (v_0 + \frac{mg}{b}) (e^{-t/\tau}) - \frac{mg}{b}$$

$$y = (v_0 + \frac{mg}{b}) \int e^{-t/\tau} dt - \int \frac{mg}{b} dt$$

$$y = (-\tau) (v_0 + \frac{mg}{b}) (e^{-t/\tau} - 1) - \frac{mg}{b} t$$

Vertical

$$\Sigma \vec{F} = m \cdot \vec{a}$$

$$m \cdot \frac{dv}{dt} = -cv^2$$

$$\frac{dv}{v^2} = \frac{-c}{m} dt$$

$$\int v^{-2} dv = \int \frac{-c}{m} dt$$

$$-\frac{1}{v} \Big|_{v_0}^{vt} = \frac{-c}{m} t$$

$$-\frac{1}{vt} + \frac{1}{v_0} = \frac{-c}{m} t$$

$$\frac{1}{vt} = \frac{1}{v_0} + \frac{c}{m} t$$

$$\frac{1}{vt} = \frac{1}{v_0} (1 + \frac{cv_0}{m} t)$$

$$\frac{1}{vt} = \frac{1}{v_0} (1 + \frac{t}{\tau})$$

$$vt = \frac{v_0}{1 + t/\tau}$$

Horizontal motion

$$\frac{dx}{dt} = \frac{v_0}{1 + t/\tau}$$

$$x - x_0 = \tau v_0 \int \frac{\frac{1}{\tau} dt}{1 + t/\tau}$$

$$x = x_0 - \tau v_0 \ln (1 + \frac{t}{\tau})$$

$$\Sigma F = m \cdot a$$

$$m \cdot \frac{dv}{dt} = mg - cv^2$$

$$\frac{dv}{dt} = g - \frac{cv^2}{m}$$

$$\frac{dv}{dt} = g (1 - \frac{cv^2}{mg})$$

$$\frac{dv}{(1 - \frac{cv^2}{mg})} = g dt$$

$$\int \frac{dv}{(1 - \frac{v^2}{v_m^2})} = \int g dt$$

$$\frac{v}{v_m} = u$$

$$dv = v_m du$$

$$\int \frac{v_m du}{(1 - u^2)} = \int g dt$$

$$v_m \int \frac{du}{(1 - u^2)} = \int g dt$$

$$v_m \arg \tan u = g t$$

$$v_m \cdot \arg \tan \frac{vt}{v_m} = g t$$

$$\frac{vt}{v_m} = \tanh \left(\frac{gt}{v_m} \right)$$

$$vt = v_m \cdot \tanh \left(\frac{gt}{v_m} \right)$$

Vertical motion

$$\frac{dy}{dt} = v_m \int \tanh \left(\frac{gt}{v_m} \right) dt$$

$$y = v_m \cdot \int \frac{\tanh \frac{gt}{v_m}}{\cosh \frac{gt}{v_m}} \cdot \frac{v_m}{g}$$

$$y = \frac{v_m^2}{g} \cdot \cosh \frac{gt}{v_m}$$